# Comparison between two different full-wave methods for the computation of nonlinear ultrasound fields in inhomogeneous and attenuating tissue

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***Abstract*—Nonlinear propagation is important in many diagnostic and therapeutic applications of medical ultrasound. The design of equipment and protocols for nonlinear modalities is facilitated by the simulation of the nonlinear ultrasound field. However, many existing simulation tools have difficulties of dealing with realistic features like tissue inhomogeneity, power law losses, or steered beams. Recently, two full-wave simulation methods for nonlinear ultrasound have been developed that are able to deal with these features. Those methods are known as the Iterative Nonlinear Contrast Source method (INCS; an integral equation method) and k-Wave (a pseudospectral time domain method). This paper assesses the accuracy of both methods by comparing their spatial and spectral results for two test configurations. In both configurations, a square piston excites a three-cycle Gaussian-modulated tone burst with a center frequency of 1 MHz and a source pressure of 750 kPa. The medium in the first configuration is homogeneous and has a speed of sound, density of mass and parameter of nonlinearity equal to that of water, and a power law attenuation with an exponent 1.5 and a magnitude of 0.75 dB/cm at 1 MHz. In the second configuration, the medium has been made inhomogeneous by putting a hollow cylinder (speed of sound equal to 1540 m/s) and a solid sphere (parameter of nonlinearity equal to 1) in the course of the radiated beam. In both cases, the results obtained with INCS and k-Wave are in excellent agreement, with maximum local differences in the order of 0.5-0.6 dB in the significant parts of the field. Because both methods are computationally quite different, it is improbable that these both suffer from the same systematic error. Hence it is established that both methods are correct and highly accurate, and are suitable tools for performing precise simulations and generating accuracy benchmarks.**

## I. Introduction

Nonlinear acoustics is of continuing interest for medical diagnostic and therapeutic ultrasound because it provides various opportunities to improve image quality [1], [2] and heat deposition [3]. To exploit these opportunities, novel imaging modalities and devices must be developed. The nonlinear nature of the involved phenomena implies that this cannot be done without accurately simulating the involved nonlinear acoustic wave fields.

A number of methods have been developed for the simulation of three-dimensional, nonlinear, pulsed acoustic fields excited by medical transducers [4], [5]. *Forward-wave methods* are the most frequently used. These start with the acoustic field at the source plane $z_0 = 0$ and subsequently march out the field over successive computational planes $z = z_0 + k\Delta z$, $k = 1, 2, 3 \ldots$ The step size $\Delta z$ may be relatively large, i.e. its size may be several wavelengths. Each step consists of separate substeps that account for the effects of diffraction, attenuation, and nonlinear distortion. Due to their nature, forward-wave methods can only deal with waves that travel away from the source. Moreover, in many cases the more restrictive assumption is being made that the field propagates almost perpendicularly to the computational planes. These facts prohibit the use of forward-wave methods in the case of strongly focused or scattered fields, for beams that considerably deviate from the $z$-axis, or for grating lobes and other wide-angle phenomena. In addition, heterogeneities are usually not dealt with.

*Full-wave methods* directly solve the relevant basic acoustic equations, often using a Finite Difference method or Finite Element method. This approach does not involve a preferred direction of propagation, and can easily deal with medium heterogeneities. A restriction of the standard implementation of these methods is that at least 10-20 grid points per smallest wavelength and per shortest period are needed. For a realistic computational domain, this easily results in computational grids that are too large to handle.

Over the last few years, two full-wave methods have been developed that do not suffer from the above restrictions. k-Wave employs a pseudospectral method in which the number of spatial grid points is largely reduced by performing the spatial derivatives in the Fourier domain. The Iterative Nonlinear Contrast Source (INCS) method is an integral equation method in which the number of spatial and temporal grid points is minimized by applying appropriate filtering in the occurring spatiotemporal convolution operation. Both methods can deal with nonlinear waves propagating in an arbitrary direction in heterogeneous media with power law losses. The only approximations that are made with both methods are related to the discretization of the computational domain.

The performance of INCS and k-Wave has been demonstrated in a number of papers [6], [7], [8], [9], [10], [11]. These papers show that INCS and k-Wave can deal with realistic models of situations encountered in nonlinear medical ultrasound, and are numerically sound. On the other hand, both methods are based on different computational principles and it is improbable that these both suffer from the same systematic error. Therefore, we think that it is interesting to quantitatively compare the results of both methods for different situations. When the results agree, we can confidently consider *both* methods to be correct and accurate. This will establish INCS and k-Wave as suitable tools for performing accurate simulations.

To perform the comparison, two configurations are defined. Both methods are used to compute the ultrasound fields in these configurations. Next, for both configurations the results are compared in the time domain and in the frequency domain.

## II. Description of methods

### A. k-Wave [6], [7], [8]

In the linear and lossless case, this method solves a set of first-order acoustic equations equivalent to

$$\frac{\partial \boldsymbol{v}}{\partial t} = \frac{-1}{\rho_0}\,\boldsymbol{\nabla} p, \tag{1}$$

$$\frac{\partial p}{\partial t} = \frac{-1}{\kappa_0}\,\boldsymbol{\nabla}\cdot\boldsymbol{v}, \tag{2}$$

where $p = p(\boldsymbol{x},t)$ is the acoustic pressure, $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x},t)$ the particle velocity, $\rho_0$ the ambient density of mass of the medium, and $\kappa_0$ its compressibility. For ease of notation, here we will only consider the one-dimensional version of these equations. Instead of using finite differences in the spatial domain, the differentiations are performed in the Fourier domain, which is arrived at by applying the Fast Fourier Transform (FFT). The differentiation of a function $f$ with respect to variable $x$ is then achieved by

$$\frac{\partial f}{\partial x} \approx F_x^{-1}\{-jk_x F_x\{f\}\}. \tag{3}$$

Here, $F_x$ and $F_x^{-1}$ are the forward and inverse FFTs, and $k_x$ is the corresponding spectral parameter. For FFTs with $N$ points, Eq. (3) can be considered equivalent to a $N$-th order finite difference scheme. When an ordinary second-order central difference scheme for the time is used and this is combined with the staggered grid approach, the resulting discrete, one dimensional equivalent of Eqs. (1) and (2) is

$$\begin{aligned}\frac{v_x^{n+1/2} - v_x^{n-1/2}}{\Delta t} &= \frac{-1}{\rho_0}\,F_x^{-1}\{-jk_x\,\vartheta\,\exp(-jk_x\Delta x/2)\\ &\qquad\times F_x\{p^n\}\},\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{p^{n+1} - p^{n}}{\Delta t} &= \frac{-1}{\kappa_0}\,F_x^{-1}\{-jk_x\,\vartheta\,\exp(jk_x\Delta x/2)\\ &\qquad\times F_x\{v_x^{n+1/2}\}\},\end{aligned} \tag{5}$$

with $\vartheta = 1$. By setting $\vartheta = \mathrm{sinc}(c_0 k\Delta t/2)$, the k-space pseudospectral method avoids the phase error that arises from the temporal finite difference. k-Wave is an implementation of the of the k-space pseudospectral method, written in Matlab and C++. k-Wave includes extensions to deal with nonlinearity, power law losses, and heterogeneity, equivalent to a generalized Westervelt equation [7], [8].

### B. INCS [9], [10], [11]

The basis of this method is the Westervelt equation

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial_t^2} = -S_{\mathrm{pr}} - \frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2} \tag{6}$$

where $c_0$ is the ambient wave speed, $S_{\mathrm{pr}} = S_{\mathrm{pr}}(\boldsymbol{x},t)$ represents the primary source (i.e. the action of the transducer), and the second term at the right hand side accounts for the nonlinear behavior, with $\beta$ being the parameter of nonlinearity. When we momentarily neglect the last term, we are just left with the linear wave equation with an explicit source term. This equation has the solution

$$p = p^{(0)} = G *_{\boldsymbol{x},t} S_{\mathrm{pr}}, \tag{7}$$

where $G = G(\boldsymbol{x},t)$ is the Green's function and $*_{\boldsymbol{x},t}$ denotes a convolution over space and time. The Green's function is the spatiotemporal impulse response of the acoustic medium, i.e. the solution of

$$\nabla^2 G - \frac{1}{c_0^2}\frac{\partial^2 G}{\partial_t^2} = -\delta(\boldsymbol{x})\,\delta(t). \tag{8}$$

When the last term of Eq. (6) is *not* neglected, we can just extend Eq. (7) with an additional term and obtain

$$p = G *_{\boldsymbol{x},t} [S_{\mathrm{pr}} + S_{\mathrm{nl}}(p)], \tag{9}$$

with the so-called nonlinear contrast source term

$$S_{\mathrm{nl}}(p) = \frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2}. \tag{10}$$

Equation (9) is an implicit solution. Explicit solutions may be obtained by using, e.g., the Neumann iterative solution

$$p^{(n)} = p^{(0)} + G *_{\boldsymbol{x},t} S_{\mathrm{nl}}[p^{(n-1)}], \qquad (n \geq 1). \tag{11}$$

Each next iteration $n$ of this scheme gives an increasingly accurate approximation to the exact nonlinear wave field. The convolutions are performed with only two grid points per wavelength, or period, of the highest frequency of interest, and are performed as multiplications in the Fourier domain. To avoid aliasing errors, the results of each iteration are subjected to a highly efficient filtering step. The scheme above describes the original INCS method [9], [10]. Later, it has been extended with additional contrast source terms that account for arbitrary loss mechanisms [11] and medium inhomogeneities [12].

## III. Comparison

We define two test configurations indicated as A and B. In both cases, the source is a square piston facing in the $z$-direction and generating a pulsed acoustical surface pressure

$$p^{\mathrm{source}} = P_0\,\exp\left[-(2t/t_w)^2\right]\,\sin(2\pi f_0 t), \tag{12}$$

in which $P_0 = 750\,\mathrm{kPa}$ is the source pressure amplitude, $f_0 = 1\,\mathrm{MHz}$ the source frequency, and $t_w = 3/f_0$ the length of the tone-burst. The medium in configuration A is homogeneous, nonlinear and lossy, and is described by a wave speed $c_0 = 1482\,\mathrm{m/s}$, a density of mass $\rho_0 = 1000\,\mathrm{kg/m}^3$, a coefficient of nonlinearity $\beta = 3.48$, and a power law attenuation coefficient $\alpha = \alpha_0\, f^b$ with $\alpha_0 = 0.75\,\mathrm{dB\,MHz}^{-b}\,\mathrm{cm}^{-}1$ and $b = 1.5$. In configuration B, the medium is made inhomogeneous by placing two contrasting objects in the medium of configuration

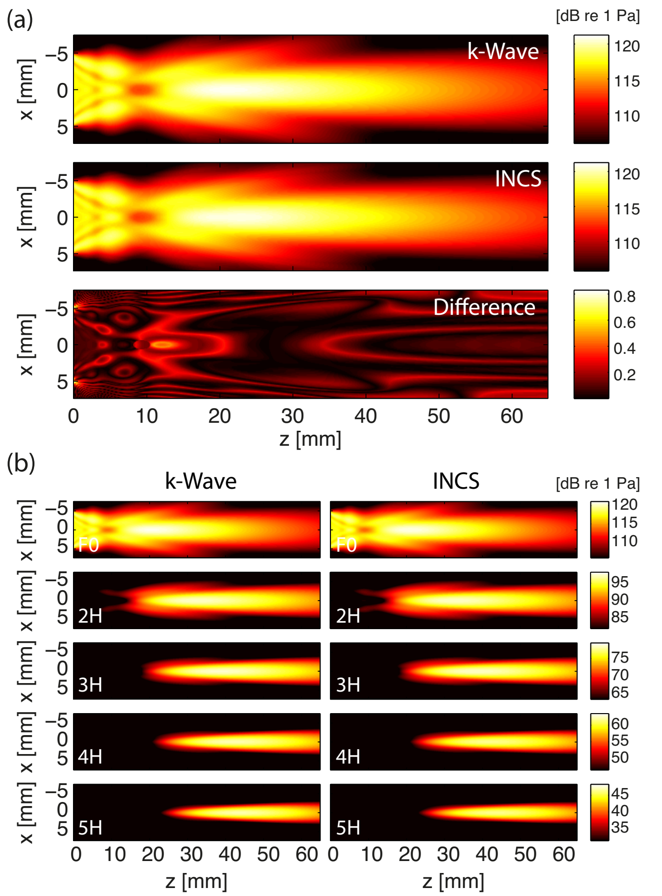


Fig. 1. Comparison between INCS and k-Wave for a homogeneous, nonlinear and lossy medium. (a) Maximum total acoustic pressure. (b) Maximum pressure of the first five harmonics. The piston transducer is located at the left side between $x = \pm 5$ mm. The pressures are given for the plane $y = 0$ mm.

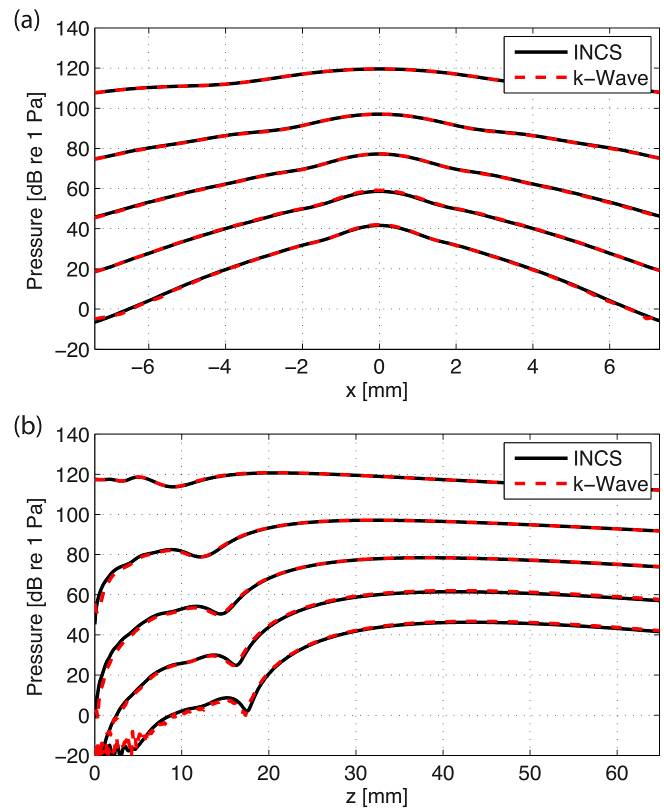


Fig. 2. Comparison between INCS and k-Wave for a homogeneous, nonlinear and lossy medium. (a) Maximum pressure of the first five harmonics along the horizontal lateral line through the peak of the wave field. (b) Maximum pressure of the first five harmonics along the beam axis.

A. These objects are a hollow cylinder with $c_0 = 1540\,\mathrm{m/s}$ and a solid sphere with $\beta = 1$ and $\alpha_0 = 1.50\,\mathrm{dB\,MHz^{-1.5}\,cm^{-1}}$. These objects are placed in the course of the ultrasound beam.

In Figs. 1 and 3 we present the distribution of the generated ultrasound fields, as computed by k-Wave and INCS for configurations A and B, in the plane $y = 0$ mm. The plots in both panels (a) refer to the maximum of the total time domain signal at each location. The plots in the panels (b) show the maximum of the time domain signal of the harmonics at each position. The harmonics are obtained by applying numerical band pass filters to the total computed time domain signals.

In Figs. 2 and 4 we show two specific cross sections of the presented harmonic beam profiles. Panels (a) of these figures show the maximum of the harmonic signals on the line parallel to the $x$-axis and through the peak of the wave field (lateral profiles), and panels (b) show the maxima on the $z$-axis (axial profiles).

In all cases, we employed $160 \times 160 \times 600$ spatial grid points with a grid point spacing of $114\,\mu\mathrm{m}$ in each Cartesian direction. At 2 spatial grid points per wavelength (Nyquist limit), this corresponds to a maximum supported frequency of 6.5 MHz for the applied background medium. For the INCS simulations, we employed a co-moving time window of 399 points for configuration A and 599 points for configuration B. At 2 temporal grid points per period at 6.5 MHz, this implies a Courant number of 1. k-Wave does not apply a co-moving time window, but for these simulations we employed 8 grid points per period at 6.5 MHz (Courant number 0.25). Further, the Neumann scheme in the INCS simulations was iterated 6 times in case A and 12 times in case B.

All figures show that there is excellent qualitative and quantitative agreement between the two methods. For the homogeneous configuration A, the maximum relative difference in the total field is on the order of $0.5$ dB, except in a very small area near the edge of the transducer, where the difference is $0.8$ dB. For the heterogeneous configuration B, the maximum relative difference in the total field is on the order of $0.6$ dB, except near the lower right corner of the computational domain and again near the edge of the transducer, where the difference is $0.8$ dB. In both configurations, the 'higher' discrepancies arise in areas where the field is actually insignificant.

## IV. Conclusion

We have demonstrated that both k-Wave and INCS are able to compute nonlinear ultrasound fields in homogeneous and heterogeneous media with tissue-realistic attenuation. In the significant part of the fields, the quantitative differences between both methods are in the order of $0.5$-$0.6$ dB at most,

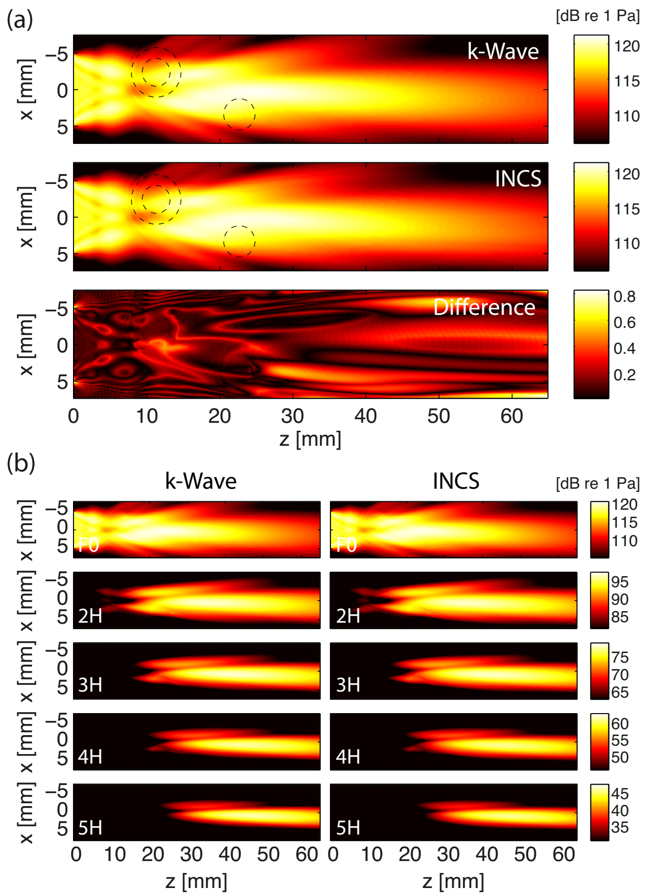


Fig. 3. Comparison between INCS and k-Wave for a heterogeneous, nonlinear and lossy medium. (a) Maximum total acoustic pressure. (b) Maximum pressure of the first five harmonics. The piston transducer is located at the left side between $x = \pm 5$ mm. The pressures are given for the plane $y = 0$ mm. The heterogeneities are indicated by dashed circles.

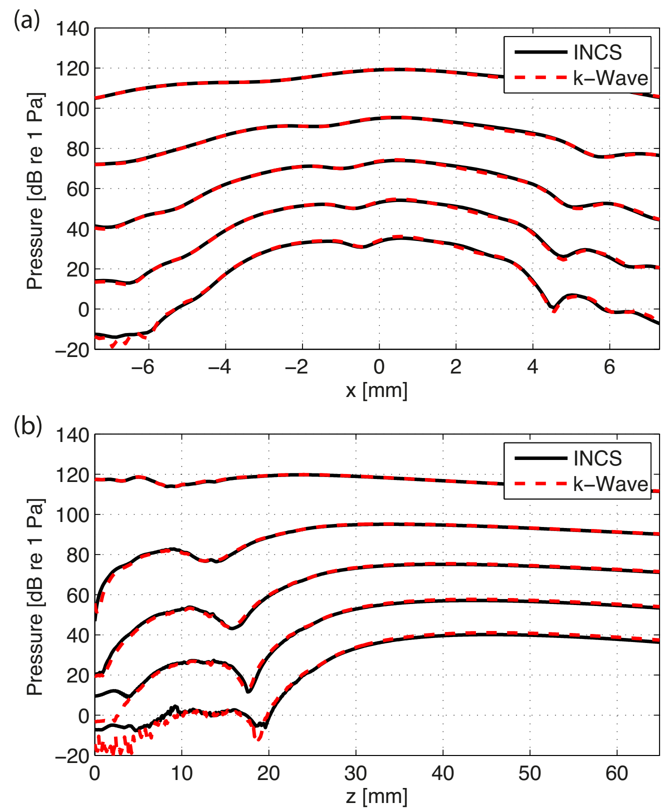


Fig. 4. Comparison between INCS and k-Wave for a heterogeneous, nonlinear and lossy medium. (a) Maximum pressure of the first five harmonics along the horizontal lateral line through the peak of the wave field. (b) Maximum pressure of the first five harmonics along the beam axis.

and in insignificant parts an difference of at most $0.8$ dB sporadically arises. Because both methods are computationally quite different, this numerical agreement gives us confidence in the correctness and accuracy of both methods. In view of this, we consider both methods suitable tools for performing precise simulations and generating accuracy benchmarks

## References

[1] M. Averkiou, D. Roundhill, and J. Powers, "A new imaging technique based on the nonlinear properties of tissues," in *2004 IEEE Ultrasonics Symposium*, 1997, pp. 1561–1566.

[2] P. van Neer, M. Danilouchkine, M. Verweij, L. Demi, M. Voormolen, A. van der Steen, and N. de Jong, "Comparison of fundamental, second harmonic, and superharmonic imaging: A simulation study," *J. Acoust. Soc. Am.*, vol. 130, pp. 3148–3157, 2011.

[3] I. Hallaj and R. Cleveland, "Fdtd simulation of finite-amplitude pressure and temperature fields for biomedical ultrasound," *J. Acoust. Soc. Am.*, vol. 105, pp. L7–L12, 1999.

[4] M. Verweij and J. Huijssen, "Computational methods for nonlinear acoustic wavefields," in *Computational methods in nonlinear acoustics: Current trends*, C. Vanhille and C. Campos-Pozuelo, Eds. Kerala, India: Research Signpost, 2011, pp. 1–19.

[5] M. Verweij, B. Treeby, K. van Dongen, and L. Demi, "Simulation of ultrasound fields," in *Comprehensive biomedical physics*, A. Brahme, Ed. Amsterdam: Elsevier, 2014, pp. 465–500.

[6] B. Treeby and B. Cox, "k-wave: Matlab toolbox for the simulation and reconstruction of photoacoustic wave fields," *J. Biomed. Opt.*, vol. 15, p. 021314, 2010.

[7] B. Treeby, J. Jaros, A. Rendell, and B. Cox, "Modeling nonlinear ultrasound propapgation in heterogeneous media with power law absorption using a $k$-space pseudospectral method," *J. Acoust. Soc. Am.*, vol. 131, pp. 4324–4336, 2012.

[8] J. Jaros, A. Rendell, and B. Treeby, "Full-wave nonlinear ultrasound simulation on distributed clusters with applications in high-intensity focused ultrasound," *arXiv:1408.4675 [physics.med-ph]*, 2014.

[9] J. Huijssen, "Modeling of nonlinear medical diagnostic ultrasound," Ph.D. dissertation, Delft University of Technology, the Netherlands, 2011.

[10] J. Huijssen and M. Verweij, "An iterative method for the computation of nonlinear, wide-angle, pulsed acoustic fields of medical diagnostic transducers," *J. Acoust. Soc. Am.*, vol. 127, pp. 33–44, 2010.

[11] L. Demi, K. van Dongen, and M. Verweij, "A contrast source method for nonlinear acoustic wave fields in media with spatially inhomogeneous attenuation," *J. Acoust. Soc. Am.*, vol. 129, pp. 1221–1230, 2011.

[12] L. Demi, M. Verweij, and K. van Dongen, "Modeling three dimensional nonlinear pressure wave fields in media with spatially varying coefficient of nonlinearity, attenuation and speed of sound," in *2012 IEEE Ultrasonics Symposium*, 2012, pp. 519–522.